L' Hospital Rule

1. Explain why we cannot use L' Hospital Rule to calculate the followings:

(a) $\lim_{x\to 0}\frac{x^2\sin\frac{1}{x}}{\sin x}$ **(b)** $\lim_{x\to\infty}\frac{x-\sin x}{x+\sin x}$ **(c)** $\lim_{x\to\infty}\frac{2x+\sin 2x}{(2x\sin x)e^{\sin x}}$

2. Evaluate:

(a) $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\cos 2x}$ **(b)** $\lim_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\cos 3x}$ **(c)** $\lim_{x\to\frac{\pi}{2}}\frac{\tan x}{\tan 3x}$

(d) $\lim_{x\to\pi}(\pi-x)\tan\frac{x}{2}$ **(e)** $\lim_{x\to 0}\left(\frac{1}{\sin^3 x}-\frac{1}{x^3}\right)$ **(f)** $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$

(g) $\lim_{x\to 0}\frac{x-x\cos x}{x-\sin x}$ **(h)** $\lim_{x\to 0}\frac{\tan x-x}{x-\sin x}$

3. Evaluate:

(a) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ **(b)** $\lim_{x\to 0}\frac{x-\tan x}{x^3}$ **(c)** $\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

(d) $\lim_{x\to 1}(x^2-1)\tan\frac{\pi x}{2}$ **(e)** $\lim_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{x}$ **(f)** $\lim_{x\to 3}\frac{\sqrt{3x}-\sqrt{12-x}}{2x-3\sqrt{19-5x}}$

(g) $\lim_{n\to\infty}\left(n^2-\frac{n}{\tan\frac{1}{n}}\right)$

4. Evaluate:

(a) $\lim_{x\to 0}\frac{x+\tan x}{\sin 2x}$ **(b)** $\lim_{x\to 0}\frac{\tan^{-1}x}{x}$ **(c)** $\lim_{x\to 0}\frac{e^{2x}-1}{\ln(1+x)}$

(d) $\lim_{x\to 0}\frac{a^x-1}{x}$, where $a>0$ **(e)** $\lim_{x\to\infty}x^3e^{-x}$ **(f)** $\lim_{x\to 0^+}\sin x\ln x$

(g) $\lim_{x\to 0}\left(\frac{1}{x}-\csc x\right)$ **(h)** $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$ **(i)** $\lim_{x\to\frac{\pi}{2}}(\tan 5x-\tan x)$

(j) $\lim_{x\to\infty}\frac{\ln(e^x+x^2)}{x^2}$ **(k)** $\lim_{x\to\infty}x^{\frac{1}{x}}$ **(l)** $\lim_{x\to 0}x^{\sin x}$

(m) $\lim_{x\to 0^+}x^x$ **(n)** $\lim_{x\to 1}x^{\frac{1}{1-x}}$